

Autumn Semester Examination 2018
Paro College of Education
Royal University of Bhutan
Paro

Module : MAT 307 (Differential Calculus)

Programme: B.Ed(S)

Level : III

Writing Time: Three Hours

Full Marks: 100

Instructions : Do not write during the first 15 minutes. Use this time for reading the questions. You will get three hours for answering the questions. Write the answers to all the questions in the answer sheets provided by the college. Read the directions to each section and to each question carefully before answering the questions. You are allowed to carry a scientific calculator *fx-82 or fx-100* beside other writing materials.

Instructions : This paper contains FIVE questions. Answer any FOUR questions. Each questions carry 25 marks. Marks for each question or sub question are given in the brackets.

Question 1

a. State and prove the condition of existence of limits. [7]

b. A function $f(x)$ defines as

$$f(x) = \begin{cases} \frac{\sin 3x}{\tan^{-1} 2x}, & \text{if } x < 0 \\ \frac{1}{2}, & \text{if } x = 0 \\ \frac{\log(1+3x)}{e^{2x}-1}, & \text{if } x > 0 \end{cases}$$

Show that $f(x)$ has hole in the graph at $x = 0$. How can you make given the function a continuous function at $x = 0$. [6]

c. Differentiate $\sqrt{\cos 3x}$ with respect to x by the delta method. [6]

d. Show that among all the rectangles with equal perimeter, the square has the largest area. [6]

Question 2

a. Find the horizontal and vertical asymptote of the function $f(x) = \frac{x^2 + 9}{3x^2 + 11x + 6}$ [6]

b. Show that the function $f(x) = \begin{cases} \frac{e^{1/x} - 1}{e^{1/x} + 1}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$ is discontinuous at $x = 0$.

Discuss the type of discontinuity. What type of graph is expected? [6]

c. If $y = x^{\sqrt{x}} + (\log x)^{\sin x}$, find $\frac{dy}{dx}$. [7]

d. Find the equations of the tangent and the normal at the point ' t ' on the curve $x = a \sin^3 t$, $y = b \cos^3 t$. [6]

Question 3

a. Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ [6]

b. If $f(x) = \begin{cases} \frac{x-4}{|x-4|} + p, & \text{if } x < 4 \\ p+q, & \text{if } x = 4 \\ \frac{x-4}{|x-4|} + q, & \text{if } x > 4 \end{cases}$ is continuous at $x = 4$, find the values of p and q . [7]

c. If $\log(x^2 + y^2) = 2 \tan^{-1} \left(\frac{y}{x} \right)$, show that $\frac{dy}{dx} = \frac{x+y}{x-y}$. [6]

d. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube. [6]

Question 4

a. Evaluate $\lim_{x \rightarrow 0} \frac{10^x - 2^x - 5^x + 1}{\sin^{-1} 2x [\log(3+x) - \log(3-x)]}$ [6]

b. Discuss the continuity of the function $f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 \\ x+1, & \text{if } x \geq 0 \end{cases}$. [6]

c. If $x = a \cos \theta + b \sin \theta$ and $y = a \sin \theta - b \cos \theta$, prove that

$$y^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 0. \quad [6]$$

d. Find the equation of tangent line to $y = 2x^2 + 7$ which is parallel to the line $4x - y + 3 = 0$. [7]

Question 5

a. Evaluate $\lim_{x \rightarrow 0} \frac{(x+y) \csc(x+y) - y \csc y}{\sin^{-1} x}$ [6]

b. Show that the function $f(x) = \begin{cases} |2x - 3| \lfloor x \rfloor, & \text{if } x \geq 1 \\ \sin \left(\frac{\pi x}{2} \right), & \text{if } x < 1 \end{cases}$ is continuous but not differentiable at the point $x = 1$. [7]

c. If $y = \frac{x \sin^{-1} x}{\sqrt{1-x^2}}$, then prove that $(1-x^2) \frac{dy}{dx} = x + \frac{y}{x}$. [6]

d. Find all the points of local maxima and minima with their corresponding values of the function $f(x) = \sin x + \frac{1}{2} \cos 2x$, where $0 \leq x \leq \frac{\pi}{2}$. [6]